University of California, Berkeley
Physics H7A Fall 1998 (*Strovink*)

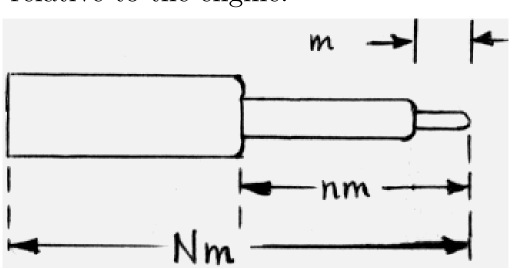
PROBLEM SET 4

1. A very flexible uniform chain of mass M and length L is suspended from one end so that it hangs vertically, the lower end just touching the surface of a table. The upper end is suddenly released so that the chain falls onto the table and coils up in a small heap, each link coming to rest the instant that it strikes the table. Find the force exerted by the table on the chain at any instant, in terms of the weight of the chain already on the table at that moment.

2. A supersonic plane of mass M has an airspeed $v = 1000$ m/sec. Its jet engine takes in 80 kg of air per sec, mixes it with 30 kg of fuel per sec, and compresses the mixture so that it ignites. The resulting hot gasses leave the engine with velocity 3000 m/sec relative to the plane. What thrust (force) does the engine deliver?

3. K&K problem 3.13 "A ski tow consists of...".

4. This problem is designed to illustrate the advantage that can be obtained by the use of multiple-staged instead of single-staged rockets as launching vehicles. Suppose that the payload (*e.g.* a space capsule) has mass m and is mounted on a two-stage rocket (see figure). The total mass – both rockets fully fueled, plus the payload – is Nm. The mass of the second-stage rocket plus the payload, after first-stage burnout and separation, is nm. In each stage the ratio of burnout mass (casing) to initial mass (casing plus fuel) is r, and the exhaust speed is V, constant relative to the engine.



(a.) Show that the velocity v gained from the first-stage burn, starting from rest and ignoring gravity, is given by

$$v = V \ln \frac{N}{rN + n(1-r)}.$$

(b.) Obtain a corresponding expression for the additional velocity u gained from the second stage burn.

(c.) Adding v and u, you have the payload velocity w in terms of N, n, and r. Taking N and r as constants, find the value of n for which w is a maximum.

(d.) Show that the condition for w to be a maximum corresponds to having equal gains of velocity in the two stages. Find the maximum value of w, and verify that it makes sense for the limiting cases described by $r = 0$ and $r = 1$.

(e.) Find an expression for the payload velocity of a single-stage rocket with the same values of N, r, and V.

(f.) Suppose that it is desired to obtain a payload velocity of 10 km/sec, using rockets for which $V = 2.5$ km/sec and $r = 0.1$. Show that the job can be done with a two-stage rocket but is impossible, however large the value of N, with a single-stage rocket.

5. A boat of mass M and length L is floating in the water, stationary; a man of mass m is sitting at the bow. The man stands up, walks to the stern of the boat, and sits down again.

(a.) If the water is assumed to offer no resistance at all to motion of the boat, how far does the boat move as a result of the man's trip from bow to stern?

(b.) More realistically, assume that the water offers a viscous resistive force given by $-kv$, where k is a constant and v is the velocity of the boat. Show that in this case one has the remarkable result that the boat should eventually return to its initial position!

(c.) Consider the paradox presented by the fact that, according to (b.), any nonzero value of k, however, small, implies that the boat ends up at its starting point, but a strictly zero value of k implies that it ends up somewhere else. How do you explain this discontinuous jump in the final position when the variation of k can be imagined as continuous, down to zero? For details, see D. Tilley, *American Journal of Physics* Vol. **35**, p. 546 (1967).

6. The Great Pyramid of Gizeh when first erected (it has since lost a certain amount of its outermost layer) was about 150 m high and had a square base of edge length 230 m. It is effectively a solid block of stone of density about 2.5 g/cc.

(a.) What is the minimum amount of work required to assemble the pyramid, if the stone is initially at ground level?

(b.) Assume that a slave employed in the construction of the pyramid had a food intake of about 1500 Cal/day (1 Cal = 4182 joules). The Greek historian Herodotus reported that the job took 100,000 slaves 20 years. What was the minimum efficiency of a slave (defined as work done divided by energy consumed)?

7. A particle of mass m, at rest at t=0, is subjected to a force $\mathbf{f}(t)$ whose magnitude at t=0 is F. This magnitude decreases linearly with time, becoming zero at time $t = T$. The direction of the force remains unchanged. What is the kinetic energy of the particle at time T?

8. A wooden block of mass M, initially at rest on a table with coefficient of sliding friction μ, is struck by a bullet of mass m and velocity v. The bullet lodges in the center of the block. How far does the block slide?